

# Quiz 1

CS 3853

July 17, 2024

Name (please print): \_\_\_\_\_

Student ID: \_\_\_\_\_

Total points: 30

1. (6 points) A compiler developer is trying to compare the designs of three machines with their respective CPIs for several instruction categories. All machines have the same instruction set.

Instruction Category	$CPI_A$	$CPI_B$	$CPI_C$	Mix
Load/Store	12	11	9	10%
Subtract	8	7	5.6	20%
Jump	5	4	7	12%
Branch	9	7	12	43%
Shift	6	5	9	30%
Other	22	12	17	50%

- Calculate the average CPI for each machine

**Solution: 2 points for each**

$$\text{Average } CPI_A = (12 \times 0.10) + (8 \times 0.20) + (5 \times 0.12) + (9 \times 0.43) + (6 \times 0.30) + (22 \times 0.50) = 20.07$$

$$\text{Average } CPI_B = (11 \times 0.10) + (7 \times 0.20) + (4 \times 0.12) + (7 \times 0.43) + (5 \times 0.30) + (12 \times 0.50) = 13.49$$

$$\text{Average } CPI_C = (9 \times 0.10) + (5.6 \times 0.20) + (7 \times 0.12) + (12 \times 0.43) + (9 \times 0.30) + (17 \times 0.50) = 19.22$$

2. (4 points) Show the truth table for the following function

$$F(A, B, C) = \overline{A}\overline{B} + \overline{A}C + \overline{B}\overline{C}$$

Can you find a simpler expression for  $F$ ? If yes, please show it.

**Solution: 4 points – 3 points for the truth table, 1 point for any reasonable simplification of minterms**

A	C	D	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

$$\text{minterms} = \overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C + \overline{A}B\overline{C} + \overline{A}BC$$

$$\overline{A}\overline{B} + \overline{A}\overline{B}C + \overline{A}B\overline{C} \quad (C + \overline{C} = 1, \text{ Complementarity law})$$

$$\overline{A}(\overline{B} + BC) + \overline{A}B\overline{C}$$

$$\overline{A}\overline{B}C + \overline{A}B\overline{C} + \overline{A}\overline{B}C + \overline{A}B\overline{C} \quad (C + \overline{C} = 1, \text{ Complementarity law})$$

$$\overline{A}\overline{B}C + \overline{A}B\overline{C} + \overline{A}\overline{B}C \quad (A + A = A, \text{ Idempotent law})$$

$$\overline{A}C + \overline{A}B\overline{C} \quad (B + \overline{B} = 1, \text{ Complementarity law})$$

It is possible to have a simpler expression

3. (10 points) Given the function  $F(A, B, C, D) = \Sigma m(1, 7, 8, 10, 12, 13, 14, 15) + d(3, 5)$ .

- Draw the Karnaugh map for the function
- Find all prime implicants
- List all essential prime implicants
- Find a minimal SOP. Is your solution unique? If not give an alternate solution.

**Solution: 10 points – 4 points for kmap, 2 points each for the rest**

$f$		$c,d$			
		00	01	11	10
$a,b$	00	0	1	-	0
	01	0	-	1	0
	11	1	1	1	1
	10	1	0	0	1

- PIs:  $\overline{AD}, BD, AB, ACD, AC\overline{D}$
- Essential PIs:  $\overline{AD}, AB, ACD, AC\overline{D}$

Minimum SOP:

$$F(A, B, C, D) = \overline{AD} + A\overline{D} + AB$$

Many others are possible

4. (10 points) Simplify the boolean expression  $F(A, B, C, D) = \Sigma m(1, 7, 8, 10, 12, 13, 14, 15) + d(3, 5)$  using the Quine-McCluskey tabular method.

**Solution: 10 points – 2 points for grouping and binary representation, 4 points for the two matching steps, 2 points for the prime implicant chart, 2 points for the reduced expression**

1. Binary presentation and grouping

Group A1	1	0001	→
	8	1000	→
	10	1010	→
Group A2	12	1100	→
	3	0011	→
	5	0101	→
Group A3	7	0111	→
	13	1101	→
	14	1110	→
Group A4	15	1111	→

2. Matching Step 1

Group B1 (A1,A2)	1,3	00-1	→
	1,5	0-01	→
	8,10	10-0	→
	8,12	1-00	→
Group B2 (A2,A3)	10,14	1-10	→
	12,13	110-	→
	12,14	11-0	→
	3,7	0-11	→
Group B3 (A3,A4)	5,7	01-1	→
	5,13	-101	→
	7,15	-111	→
	13,15	11-1	→
	14,15	111-	→

## 3. Matching Step 2

Group C1	1,3,5,7	0--1	✓
(B1,B2)	8,10,12,14	1--0	✓
Group C2	12,13,14,15	11--	✓
(B2,B3)	5,7,13,15	-1-1	✓

## 4. Prime Implicant Chart

PIs\Minterms	1	7	8	10	12	13	14	15	a,b,c,d
1,3,5,7	X	X							0--1
8,10,12,14			X	X	X		X		1--0
12,13,14,15					X	X	X	X	11--
5,7,13,15		X				X		X	-1-1

5. Reduced Expression  $F(A, B, C, D) = \overline{A}D + A\overline{D}$