On the Discrete Unit Disk Cover (DUDC) Problem

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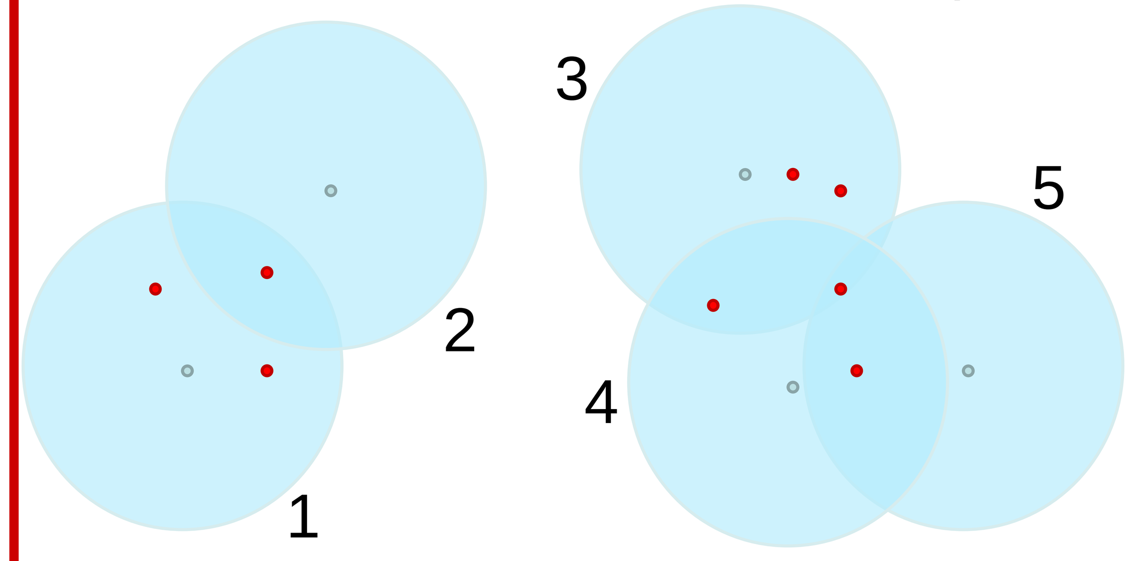
Problem Definition

Given ***m*** unit disks ***D*** and ***n*** points ***Q*** in the plane, the discrete unit disk cover problem is to select a minimum subset of the disks to cover the points



Solution – 1, 3, 4 or 1, 3, 5

Motivation

- Selecting locations for wireless servers from a set of candidate locations to cover a set of wireless clients.
- Selecting a set of weather radar antennae to cover a set of cities.
- Selecting locations for anti-ballistic defenses from a set of candidate locations to cover strategic sites.

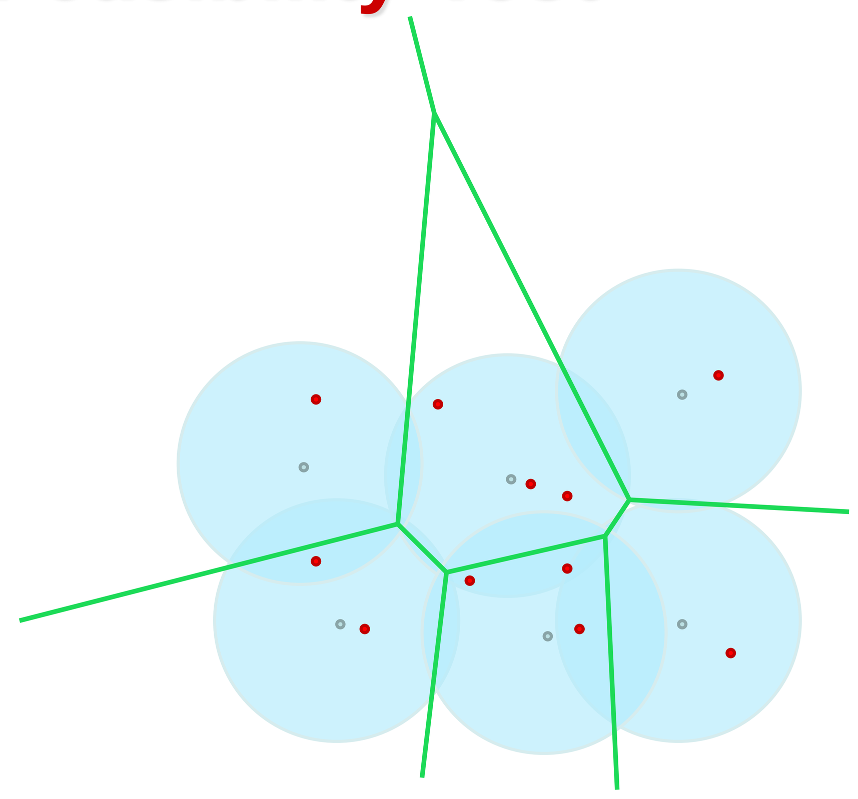
Previous Result

- **22**-approximation in **O(m²n⁴)** time
  - Claude et al., 2010
- **(1+ε)**-approximation
  - Mustafa & Ray, 2009
  - **O(m²⁵⁷n)** time for a **2**-aproximation solution.

Our Result

- **18**-approximation in **O(mn + m log m + n log n)** time

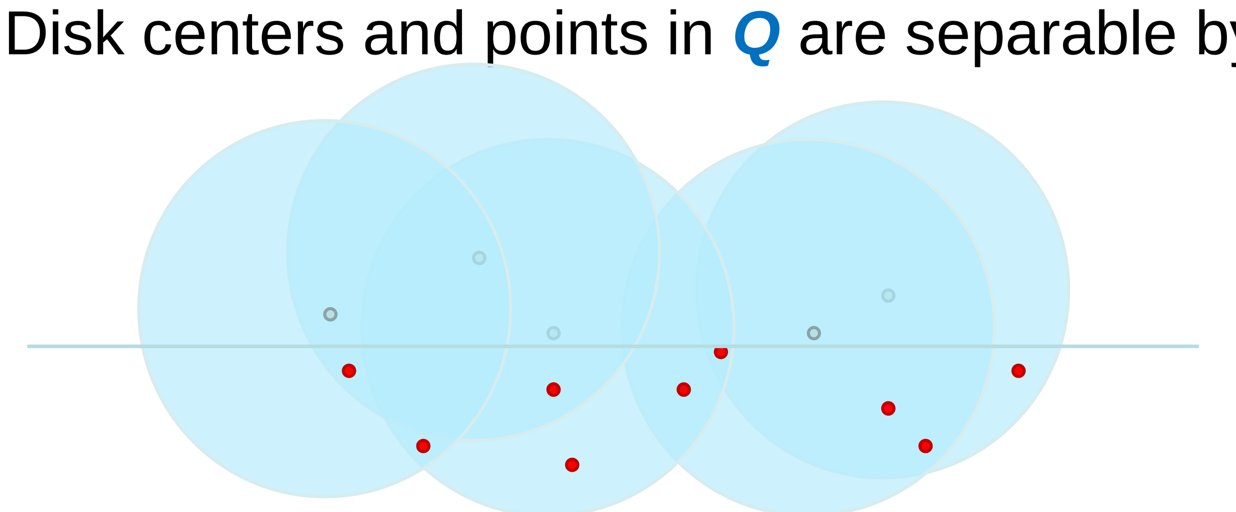
Feasibility Test



Feasibility test can be done in **O(m log m + n log m)** time.

Restricted Line-Separable DUDC (RLSDUDC)

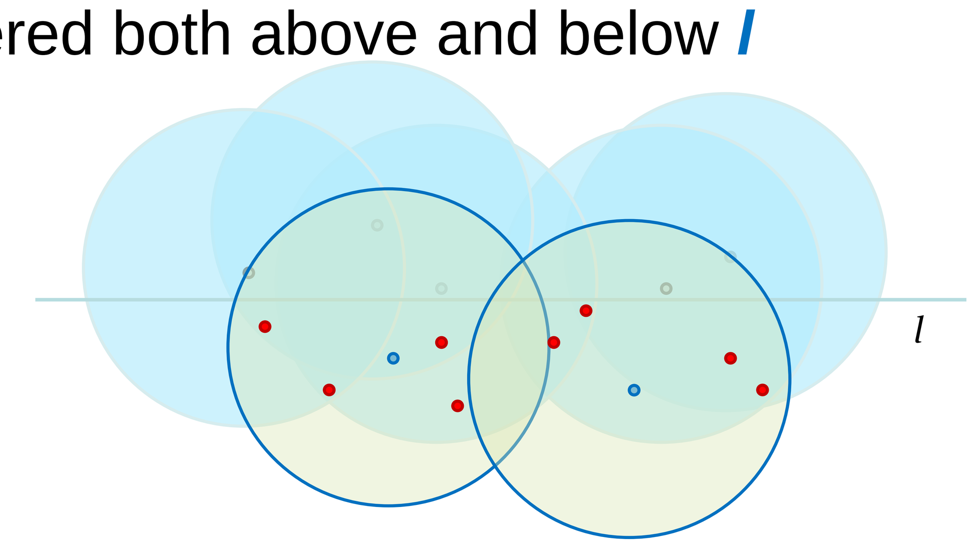
Disk centers and points in ***Q*** are separable by a line



[Claude et al., 2010] **RLSDUDC** problem can be solved optimally in **O(mn+ n log n)** time.

Line-Separable DUDC (LSDUDC)

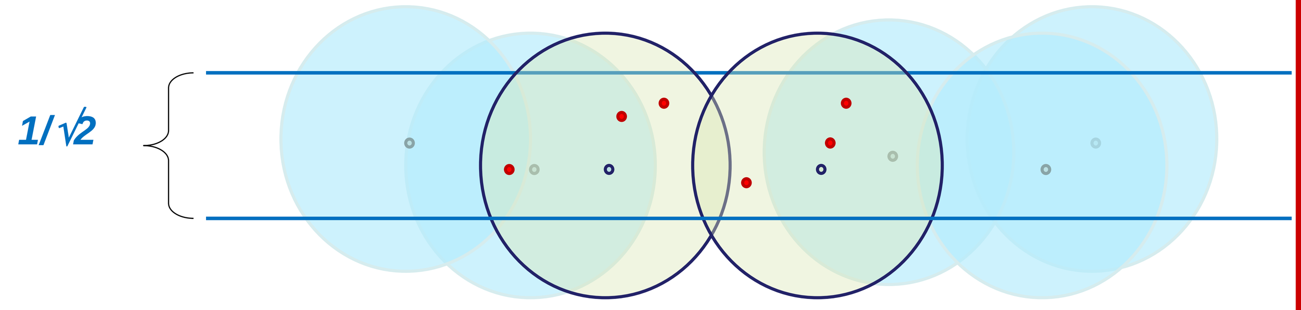
All points ***Q*** on one side of the line ***l***, and disks ***D*** centered both above and below ***l***



[Claude et al., 2010] **LSDUDC** problem has **2**-approximation result in **O(mn + n log n)** time

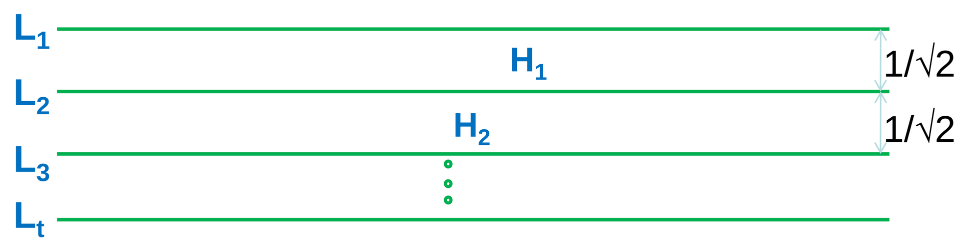
Within Strip DUDC (WSDUDC)

All points in ***Q*** and center of the disks in ***D*** are within a horizontal strip of width ***1/√2***



WSDUDC problem has a **6**-approximation algorithm with running time **O(mn + n log n)**

Outside Strip DUDC (OSDUDC)

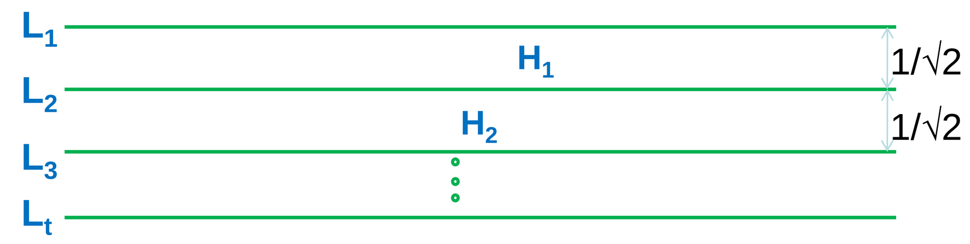


For each i = 1, 2, … t
- Compute $\mathbf{Q_i \subseteq Q}$ such that each point of $\mathbf{Q_i}$ is covered by disks centered above the line $\mathbf{L_i}$
- Run **LSDUDC** algorithm on $\mathbf{Q_i}$ based on the line $\mathbf{L_i}$, Set $\mathbf{Q = Q - Q_i}$

For each i = t, t-1, … 1
- Compute $\mathbf{Q_i \subseteq Q}$ such that each point of $\mathbf{Q_i}$ is covered by disks centered below the line $\mathbf{L_i}$
- Run **LSDUDC** algorithm on $\mathbf{Q_i}$ based on the line $\mathbf{L_i}$, Set $\mathbf{Q = Q - Q_i}$

- **OSDUDC** problem has a **12**-approximation algorithm with running time **O(mn + n log n)**

DUDC Algorithm



- Divide the region into horizontal strips of width **1/√2**
- For each horizontal strip $\mathbf{H_i = [\, L_i, L_{i+1} \,]}$
  - find the points set $\mathbf{Q_i \subseteq Q}$ which are covered by the disks centered in the strip $\mathbf{H_i}$ **only**
  - Run **Within Strip Algorithm** for the strip $\mathbf{H_i}$
- Run **Outside Strip Algorithm**

- The **DUDC** algorithm produces an **18**-approximation result in **O(mn + n log n)** time.

Conclusion

We have designed a discrete unit disk cover (**DUDC**) algorithm that provide an **18**-approximation solution in **O(mn +m log m + n log n)** time. The previous approximation factor was **22** with running time **O(m²n⁴)**; for **m = |D|**, **n = |Q|**.

Future work:
- Can the approximation factor be improved further along these lines?
- Can the running time be further improved?

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